Onset time for energy oscillations in a two-dimensional trapped Bose gas excited by a red laser potential

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We explore the energy dynamics of a two dimensional (2D) harmonically trapped Bose-Einstein condensate inside a box potential, excited by a moving red-detuned laser potential (RDLP). For an RDLP velocity v less than a critical value v_c, energy oscillations are observed to begin simultaneously with the motion of the RDLP. For $v > v_c$, these oscillations are delayed through a transient in the energy. At the end of the delay time t_{ons}, the energy oscillations are regenerated again, and t_{ons} is found to depend on v through universal relations for two cases: one for an RDLP depth sufficient to break off a BEC fragment, another for a depth insufficient to split the BEC. In the case of splitting, t_{ons} exactly equals the time it takes the BEC fragment (dragged by the RDLP trough) to reach the hard wall of the box potential.

a. Introduction: The investigation of a Bose-Einstein condensate (BEC) excited by an obstacle produced by a laser beam has received considerable theoretical and experimental interest in the last few years [1–16]. So far, mostly repulsive obstacles have been created experimentally by a blue-detuned laser beam [9–16]. Recently, however, a new method has been suggested to create attractive obstacles using a red-detuned laser beam instead [17]. In the latter, vortex dipole generation was shown to be possible with two red-detuned Gaussian beams. The success of using a red-detuned laser for the nucleation of vortices as in Aioi *et al.* [17], and the lack of literature therein, provided the main motivation for this work. In another quest, we were motivated to search for a universal behavior (UB) in our system, as UBs have been demonstrated earlier in trapped BECs [15, 18–22]. For example, Raman *et al.* [15] revealed a universal behavior connecting the thermal fraction of the condensate with the velocity of the laser. We then set out to search for a UB connecting the onset time for solitonic nucleation and the velocity of a moving attractive obstacle created by a red-detuned laser potential (RDLP).

Our chief goal in this Letter, then, is to demonstrate the presence of two UBs connecting the "onset time" t_{ons} for solitonic excitations with the velocity of the moving laser beam, both for a shallow and an RDLP. The t_{ons} is recorded in the transient stage of the simulation [23], from the start of laser motion until the energy begins to display an oscillatory behavior. As we have outlined earlier [23], and in correspondence to Parker *et al.* [24], the energy of solitons oscillates with time. In addition, we show that a deep RDLP suppresses soliton excitations. To the best of our knowledge, our current findings pertaining to the energy dynamics in a trapped BEC excited by an RDLP have not been reported earlier.

In our previous publication [23], we have demonstrated that a blue-detuned laser potential (BDLP) moving inside a harmonic trap plus hard-wall box-potential (HWBP) boundaries can lead to fascinating self-interfering matter wave patterns. In this work, we invert the "sign" of the laser potential (LP) to make it attractive and investigate the dynamics along the same lines as in our Ref.[23]. We follow the method of splitting and merging a number of BEC fragments [14, 25–28]. For example, Scherer *et al.* [14] successfully observed vortex formation by splitting a harmonically trapped BEC into three sections using a blue-detuned laser, and then allowing those sections to merge. Chang *et al.* [28] has demonstrated experimentally, using a BDLP, that solitons and dispersive shock waves (DSWs) can be produced by merging and splitting of BECs, respectively. DWs were also produced by merging BECs using a RDLP. By using a "deep" RDLP well, we separate off a BEC fragment from the main BEC and move it toward the HWBP boundary. Once it is reflected from the hard wall, after the RDLP leaves the trap, it merges again with the central part of the BEC. In essence, another goal of this Letter is to motivate experiments which explore the dynamics of a trapped BEC excited by an RDLP.

Our key results are (i) t_{ons} depends on the RDLP velocity v only, and is independent of RDLP width and depth and independent of the coupling strength $\mathcal{N}$ between the bosons, i.e., it displays universal behavior. Two universal behaviors for the t_{ons} vs v were found as follows: a) for an RDLP well deep enough to split the BEC, t_{ons} was found to be exactly equal to the time it takes the separated fragment to reach the HWBP boundary from the BEC center, that is $t_{ons} = 10/v$; b) for a shallow RDLP, insufficient to split the BEC, a different universal behavior was obtained for t_{ons}, namely $t_{ons} = 3.3/v$. We verified [i(a)] by fitting a classical curve for the relationship between t_{ons} and v, which is $t_{ons} = d/v$, where $d = 10$ is the distance from the center of the trap to the HWBP boundary. (ii) An RDLP suppresses solitonic excitations, since before the separated BEC fragment reaches the HWBP boundary, no oscillations appear in the energy dynamics.

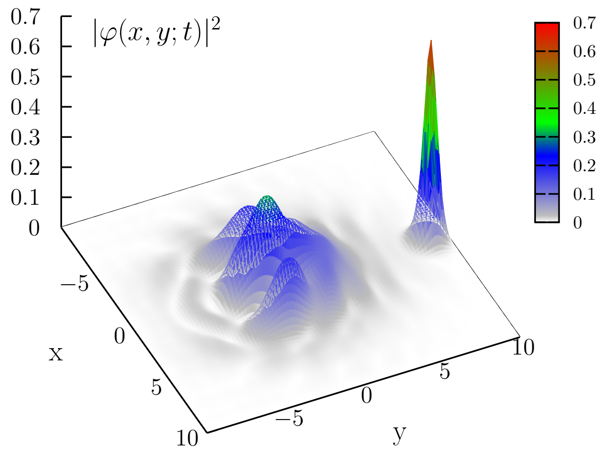

FIG. 1: Two dimensional (2D) density $|\varphi(x,y;t)|^2$ at $t = 4.5$. The system is a hard core Bose gas in a 2D harmonic trap plus box-potential boundaries [23] excited by a red-detuned laser potential with $\mathcal{N} = 10$, $A = -30$, $\beta = 4$, $v = 2$, and $\kappa = 1$. A is in units of $\hbar\omega_{ho}$, β in $(a_{ho})^{-2}$, v in a_{ho}, $t = \tau\omega_{ho}$ is unitless, $\mathcal{N}$ is in units of $(\sqrt{2}a_{ho}^2)^{-1}$, and $|\varphi(x,y;t)|^2$ is in $(a_{ho})^{-2}$. x and y are in units of a_{ho}.

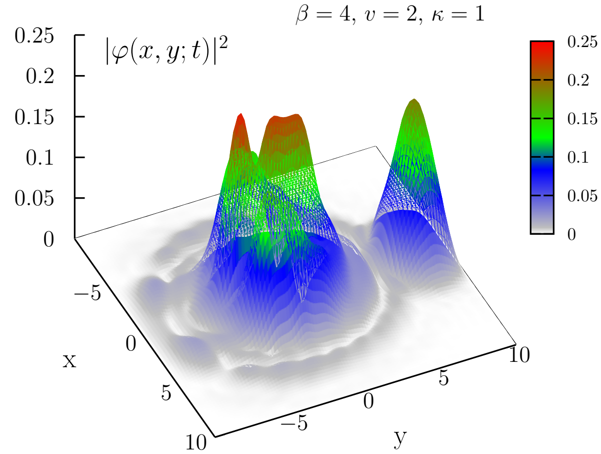

FIG. 2: As in Fig. 1; but for $t = 5.4$. The units are as in Fig. 1.

b. Method: The model we use is the same as in our previous work [23], except that now the LP is a trough instead of a barrier. The split-step Crank Nicolson method [29] is applied to solve the 2D time-dependent Gross-Pitaevskii equation (TDGPE) [Eq.(1) in Ref.[23]]. The energy $E(t)$ (function of time) is computed using Eq.(7) in Ref.[23]. The coupling constant $\mathcal{N}$, LP depth A, LP velocity v, inverse width of the LP β, total energy of the system E, and time t_{ons}, all have the same units as before [23]: The lengths and energies are in units of the trap $a_{ho} = \sqrt{\hbar/(2m\omega_{ho})}$ and $\hbar\omega_{ho}$, respectively. Hence, A is in units of $\hbar\omega_{ho}$, β in $(a_{ho})^{-2}$, v in a_{ho}, $t = \tau\omega_{ho}$ is unitless, $\mathcal{N}$ is in units of $(\sqrt{2}a_{ho}^2)^{-1}$, and the density $|\varphi(x,y;t)|^2$ is in $(a_{ho})^{-2}$. We also use a spherically symmetric harmonic trap. The atoms are attracted to the well of the RDLP, thereby creating a density peak as the atoms fill the trough. As the RDLP well moves, and for large enough $|A|$, larger than a critical value $A_c > 0$, it separates off a fragment of the BEC and carries it away from the main part. As before, our system is cut off by HWBP boundaries [23], but the RDLP can continue its motion and leave the boundary. As the fragment reaches this boundary, it bounces off and returns in the opposite direction.

c. Results and Discussion: Figures 1 and 2 display the 2D BEC densities at different times $t = 4.5$ and 5.4, respectively. The system is a trapped BEC in an isotropic 2D harmonic trap with HWBP boundaries, as in Ref.[23], with interaction strength $\mathcal{N} = 10$, potential depth $A = -30$, inverse width $\beta = 4$, velocity $v = 2$, and $\kappa = 1$. In Fig. 1, the separated BEC fragment dragged by the RDLP collides with the hard wall at $y = 10$ and $t = 4.5$. In Fig. 2, the BEC fragment has bounced off the wall and is on its way back to the center of the trap. The RDLP has already left the HWBP. Since the BEC fragment is no longer trapped by the RDLP, it increases in width and declines in amplitude as it merges into the central BEC. By monitoring the energy dynamics of this system in the transient stage of the simulation, one observes an oscillatory pattern indicative of soliton excitations. This is demonstrated in Fig. 3 which displays the energy as a function of time, $E(t)$, for a system with $\mathcal{N} = 10$, $A = -30$: (open triangles) $v = 0.3$, $\beta = 2$; (open squares) $v = 0.4$, $\beta = 2$; and (open circles) $v = 2$, $\beta = 4$. These oscillations are similar to those in Figs. 11-13 of our previous article [23], where a BDLP was used. However, whereas in the case of a BDLP the energy begins to oscillate once the BDLP is moved, in the case of an RDLP the energy oscillations suffer a delay for a certain time if the velocity of the RDLP exceeds a critical value v_c. Thus if we begin with a small velocity $v = 0.3$ and $\beta = 2$, the energy oscillations begin simultaneously with the motion of the RDLP, and a regular oscillatory pattern is observed (open triangles). On increasing v slightly to 0.4 for the same β, these oscillations are damped significantly, and delayed from $t = 0$ to $t = t_{ons} = 22.5$, after which they are reenergized (open squares). (The arrows in the figures indicate where the oscillations begin after they are delayed.) Consequently, then, there is a critical velocity, v_c, between $v = 0.3$ and $v = 0.4$ at which the energy oscillations begin to suffer a time delay. This v_c depends on the parameters of the system. However, the precise determination of v_c would require a substantial number of runs, and we therefore suffice with an estimate of v_c. It is also possible, that the system for $v = 0.4$ is in a transitional state. In comparison, for $v = 2$ and a different $\beta = 4$ (open circles), the oscillations begin at a time $t_{ons} = 4.3$ and decay

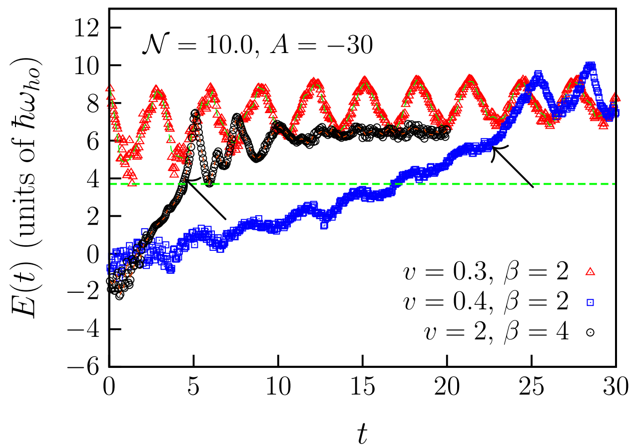

FIG. 3: Energy dynamics $E(t)$ for the same systems as that in Fig. 1; but for $v = 0.3$, $\beta = 2$ (open triangles); $v = 0.4$, $\beta = 2$ (open squares); and $v = 2$, $\beta = 4$ (open circles). The arrow indicates the onset time for solitonic nucleation, t_{ons}. $E(t)$ is in units of $\hbar\omega_{ho}$ and the rest of the units are as in Fig. 1.

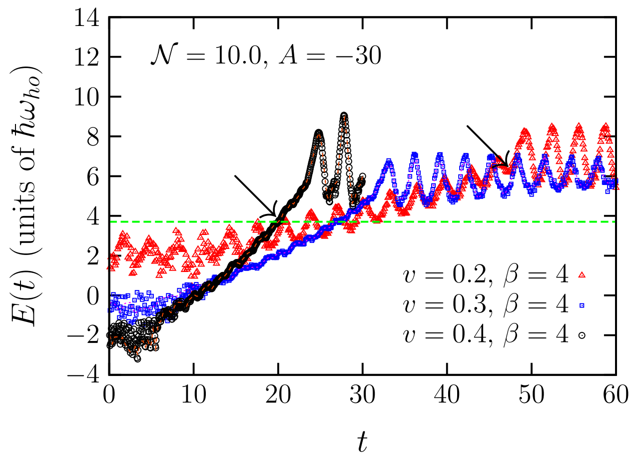

FIG. 4: As in Fig. 3; but for $v = 0.2$, $\beta = 4$ (solid triangles); $v = 0.3$, $\beta = 4$ (open squares); and $v = 0.4$, $\beta = 4$ (open circles). $E(t)$ is in units of $\hbar\omega_{ho}$ and the rest of the units are as in Fig. 1.

afterwards for $t \gtrsim 10$. The decay in these oscillations signals that upon a merger of the two separated BECs, the solitons become unstable and decay into vortex rings in the high density buldge formed during the merger [28]. Likewise, Raman *et al.* [15] found that above a critical velocity the superflow around a BDLP becomes unstable against the creation of quantized vortex lines. They also reported evidence for the existence of a critical velocity, where the BEC gas enters into a new dissipative regime.

For $v > v_c$, the initial energy of the system is negative, because for the parameters chosen the negative potential-energy of the RDLP outweighs the combination of kinetic, interaction, and trapping energies. As time progresses, $E(t)$ increases and becomes positive, as the moving obstacle transfers energy to the system in the initial stages of the simulation [30]. Eventually, $E(t)$ displays a transient similarly to the one reported earlier by Law *et al.* [30] for a repulsive potential. However, whereas in Ref.[30] the system is a homogeneous BEC, where the energy stabilizes after the transient, in our case the transient is followed by oscillations for the red-laser case. Thus, the principle feature in Fig. 3 above, is that a transient is also observed using an RDLP. In addition, according to Jackson *et al.* [5], the creation of phonons or vortices increases the energy of the condensate, which plays a role in the rise of the energy in the transient regime. For a repulsive-obstacle velocity lower than a critical one v_c, energy is transferred to the system via phonon excitations; for a velocity larger than v_c, energy is transferred via vortex formation. The oscillations in the energy following the end of the transient are, as has been outlined earlier [23] due to soliton/vortex excitations. Therefore, an important finding here is that the RDLP suppresses the solitons until the RDLP leaves the HWBP, as manifested by the transient in the energy. This is in stark contrast to the BDLP case which never suppresses solitons. After $t > t_{ons}$, the reflection of the separated BEC fragment off the HWBP boundary creates a shock wave in the system, which excites solitons, as the BEC fragment temporarily acts like a BDLP obstacle by itself, until it merges again with the central BEC part. Prior to this reflection, the RDLP peculiarly does not generate these excitations. The two BEC fragments "communicate" together by a shock wave, and cause these oscillations to arise. The backflow around this BEC fragment is also a way in which the two BECs can communicate together [31]. The effects of drag due to the interaction of the obstacle with the surrounding fluid have been explored earlier [5, 32, 33], and we shall explore these effects in a future extended version of this Letter. Next to this, when the RDLP leaves the system, the effective interaction jumps from being net attractive to net repulsive. This is, as if one "quenches" the Hamiltonian "upwards" by a sudden reduction of the attractive forces. Due to the latter quench, the system produces topological defects.

Figure 4 is the same as Fig. 3, except for a system with $v = 0.2$, $\beta = 4$ (open triangles), $v = 0.3$, $\beta = 4$ (open squares), and $v = 0.4$, $\beta = 4$ (open circles). Essentially, the same features are observed as in Fig. 3: a critical behavior between $v = 0.2$ and 0.3, and a reduction in the "onset time" with an increase in v to 0.4.

Next, we explore the possibility for the presence of universal behavior in the BEC, such as the ones reported earlier [15, 18–22]. First, by changing v and keeping A and β fixed, we determined t_{ons} as a function of v ($> v_c$) for $A = -30$, $\beta = 4$, and three different values of $\mathcal{N}$ displayed in the top frame of Fig. 5: (solid squares) $\mathcal{N} = 0$; (open circles) $\mathcal{N} = 10$; (solid triangles) $\mathcal{N} = 20$; (solid line) "fit" using classical formula $f(v) = d/v$, where $d = 10$ is the distance between the center of the trap

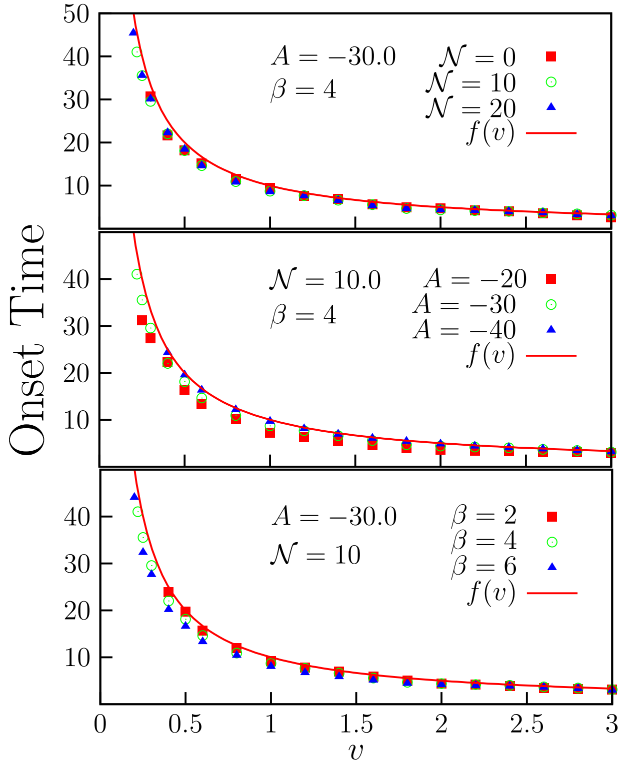

FIG. 5: Onset time for solitonic nucleation t_{ons} versus the laser velocity v for the same system as in Fig. 1 with various parameters. Here A is deep enough to split the BEC into two fragments. Top frame is for fixed $A = -30$, $\beta = 4$ and different $\mathcal{N}$: (solid squares) $\mathcal{N} = 0$; (open circles) 10; (solid circles) 20; (solid line) fit $f(v) = d/v$. Middle frame is for fixed $\mathcal{N} = 10$, $\beta = 4$ and different depths A: (solid squares) $A = -20$; (open circles) -30; (solid circles) -40. Bottom frame is for $A = -30$, $\mathcal{N} = 10$ and different β: (solid squares) $\beta = 2$; (open circles) 4; (solid circles) 6. Units are as in Fig. 1

and the hard wall (see Figs. 1 and 2). Second, the middle frame of Fig. 5 is as in the top frame; but for different amplitudes A and fixed $\mathcal{N} = 10$ and $\beta = 4$: (solid squares) $A = -20$; (open circles) $A = -30$; (solid triangles) $A = -40$; (solid line) $f(v)$. The bottom frame of Fig. 5 is for fixed $\mathcal{N} = 10$ and $A = -30$, but different widths β: (solid squares) $\beta = 2$; (open circles) $\beta = 4$; (solid triangles) $\beta = 6$; (solid line) $f(v)$. The basic feature in all these figures, is that for values of $|A|$ high enough to separate off a BEC fragment as in Fig. 1, the "onset time" for energy oscillations (solitonic nucleation) is independent of the parameters of the system. This demonstrates a universal behavior for the onset time as a function of the velocity v of the RDLP, which is almost identical to the classical $f(v) = d/v$. Of course, it may seem obvious that the time it takes the RDLP to reach the HWBP boundary is simply d/v; but the significant feature here is that the energy oscillations begin once the BEC fragment dragged by the RDLP hits the

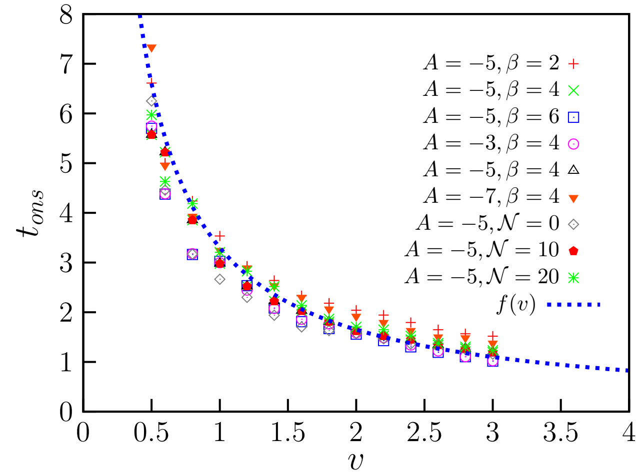

FIG. 6: As in Fig. 5; but for various parameters A, β, and $\mathcal{N}$ as indicated in the figure. Here, A is not deep enough to split the BEC into two fragments. Units are as in Fig. 5.

HWBP wall. This is also verified by the classical curve $f(v)$. The peculiar feature in Fig. 5, bottom frame, is that even though for a lower β the width of the obstacle is larger, it takes the same time to initiate energy oscillations as for a narrower one with larger β. The peculiarity arises because it takes more time for a wide obstacle to "completely" leave the HWBP trap than a narrow obstacle, as measured relative to the HWBP wall.

Finally, we display in Fig. 6 the behavior of t_{ons} with v for low enough $|A| < A_c$, such that the RDLP is not strong enough to separate off a BEC fragment. The BEC dynamics in this case is totally different. Here $|A|$ was not deep enough to suppress solitons for times comparable to Fig. 5. By changing $|A|$, $\mathcal{N}$, and β, we again observe (as labelled from cross to star) another universal behavior for t_{ons}, obeying $t_{ons} = 3.3/v$. Consequently, there are two universal behaviors for t_{ons}, one for $|A| > A_c$ at which a BEC fragment is separated, and one for $|A| < A_c$ for which the RDLP is not strong enough to separate a fragment!

d. Conclusions: In summary, then, we have explored the energy dynamics of a harmonically trapped 2D BEC inside a HWBP excited by an RDLP. In general, it was found that an attractive obstacle leads to peculiar features in the energy dynamics, not observed earlier in the case of a repulsive obstacle [23]. It was found, that the behavior of t_{ons} with the velocity of the RDLP $v > v_c$ displays two universal behaviors according to whether the RDLP is deep or shallow. We believe that this research will provide a new direction in the investigation of trapped BECs excited by a red-laser potential, especially since there seems to be a lack in the theoretical and experimental investigations applying the RDLP in trapped BECs.

[1] J. S. Stießberger and W. Zwerger, Phys. Rev. A **62**, 061601(R) (2000).
[2] K. Fujimoto and M. Tsubota, Phys. Rev. A **82**, 043611 (2010).
[3] K. Fujimoto and M. Tsubota, J. Low. Temp. Phys. **162**, 307 (2010).
[4] K. Fujimoto and M. Tsubota, Phys. Rev. A **83**, 053609 (2011).
[5] B. Jackson, J. F. McCann, and C. S. Adams, Phys. Rev. A **61**, 051603(R) (2000).
[6] Abdelaziz Radouani, Phys. Rev. A **70**, 013602 (2004).
[7] B. M. Caradoc-Davies, R. J. Ballagh, and K. Burnett, Phys. Rev. Lett. **83**, 895 (1999).
[8] B. M. Caradoc-Davies, R. J. Ballagh, and P. B. Blakie, Phys. Rev. A **62**, 011602(R) (2000).
[9] T. W. Neely, E. C. Samson, A. S. Bradley, M. J. Davis, and B. P. Anderson, Phys. Rev. Lett. **104**, 160401 (2010).
[10] P. Engels and C. Atherton, Phys. Rev. Lett. **99**, 160405 (2007).
[11] R. Onofrio, C. Raman, J. M. Vogels, J. R. Abo-Shaeer, A. P. Chikkatur, and W. Ketterle, Phys. Rev. Lett. **85**, 2228 (2000).
[12] K.W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000).
[13] C. Raman, J. R. Abo-Shaeer, J. M. Vogels, K. Xu, and W. Ketterle, Phys. Rev. Lett. **87**, 210402 (2001).
[14] David R. Scherer, Chad N. Weiler, Tyler W. Neely, and Brian P. Anderson, Phys. Rev. Lett. **98**, 110402 (2007).
[15] C. Raman, M. Köhl, R. Onofrio, D. S. Durfee, C. E. Kuklewicz, Z. Hadzibabic, and W. Ketterle, Phys. Rev. Lett. **83**, 2502 (1999).
[16] K.W. Madison, F. Chevy, V. Bretin, and J. Dalibard, Phys. Rev. Lett. **86**, 4443 (2001).
[17] Tomohiko Aioi, Tsuyoshi Kadokura, Tetsuo Kishimoto, and Hiroki Saito, Phys. Rev. X **1**, 021003 (2011).
[18] Wenxian Zhang, Z. Xu, and L. You, Phys. Rev. A **72**, 053627 (2005).
[19] I. Khan and B. Gao, Phys. Rev. A **73**, 063619 (2006).
[20] A. D. Jackson, G. M. Kavoulakis, and E. Lundh, Phys. Rev. A **69**, 053619 (2004).
[21] N. Didier, A. Minguzzi, and F. W. J. Hekking, Phys. Rev. A **79**, 063633 (2009).
[22] Markus Holzmann, Maguelonne Chevallier, and Werner Krauth, Phys. Rev. A **81**, 043622 (2010).
[23] Roger R. Sakhel, Asaad R. Sakhel, and Humam B. Ghassib, Phys. Rev. A **84**, 033634 (2011).
[24] N. G. Parker, N. P. Proukakis, and C. S. Adams, Phys. Rev. A **81**, 033606 (2010).
[25] M. R. Andrews, C. G. Townsend, H.-J. Miesner, D. S. Durfee, D. M. Kurn, W. Ketterle, Science **275**, 637 (1997).
[26] R. Carretero-González, B. P. Anderson, P. G. Kevrekidis, D. J. Frantzeskakis, and C. N. Weiler, Phys. Rev. A **77**, 033625 (2008).
[27] R. Carretero-González, N. Whitaker, P. G. Kevrekidis, and D. J. Frantzeskakis, Phys. Rev. A **77**, 023605 (2008).
[28] J. J. Chang, P. Engels, and M. A. Hoefer, Phys. Rev. Lett. **101**, 170404 (2008).
[29] P. Muruganandam and S. K. Adhikari, Computer Physics Communications **180**, 1888 (2009).
[30] C. K. Law, C. M. Chan, P. T. Leung, and M.-C. Chu, Phys. Rev. Lett. **85**, 1598 (2000).
[31] H. B. Ghassib and S. Chatterjee, Z. Phys. B - Cond. Matt. **51**, 93 (1983).
[32] Nicolas Pavloff, Phys. Rev. A **66**, 013610 (2002).
[33] B. Jackson, J. F. McCann, and C. S. Adams, Phys. Rev. A **60**, 4882 (1999).